IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #3

VECTORS & MATRICES

Name: Elizabeth Kot (10 Trust)

Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, December 14, 2012** Time allowed: **95 mins**

Student's **Performance** in Different **Criterion**			
B	4	**D**	2

cll

INSTRUCTIONS:

- Read the instructions for all questions carefully.
- Show all work, steps and proper units.
- Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- Write in **PENCIL**.
- **NOT** allowed to use any **electronic devices**, such as translators.
- Allowed to use **GDC**.

ASSESSMENT:

- Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- This task assesses Criteria **B & D** considering ALL the questions.

Criterion B: INVESTIGATING PATTERNS **(ONLY APPLICABLE TO QUESTION IN PART 1)**

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-8)
1–2 **Do Maths**	The student performs appropriate calculations (in (a) to (c), in (g), to (i), and in (k) to (m)) in order to recognize simple patterns.	The student **applies, with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns.	
3–4 **General Rule**	The student correctly solves (d) and (e) and suggests general rules in parts (f) and (n).	The student • **selects and applies** mathematical problem-solving techniques to recognize patterns, and • **suggests** relationships or general rules.	**Teacher's Final Grade**
5–6 **Test it**	The student describes relationships (in (j) and (n)) mathematically, and connects the various relationships. If a student has drawn conclusions consistent with their findings in part 2 (g), credit ***may*** be given here.	The student • **selects and applies** mathematical problem-solving techniques to recognize patterns, • **describes** them as relationships or general rules, and • **draws conclusions** consistent with findings.	(0-8)
7–8 **Prove it**	The student is able to correctly prove mathematically all the relationships and rules seen in the task, and is successful in (j) and (n).	The student • **selects and applies** mathematical problem-solving techniques to recognize patterns, • **describes** them as relationships or general rules, • **draws conclusions** consistent with findings, and • **provides justifications or proofs.**	

Criterion D: **REFLECTION (only applicable to question in Part 2).**

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2	The student's answers to questions (a) and (b) describe the meaning of his/her findings in real life.	The student **attempts to explain** whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life.
3–4	The student's answers to questions (c) (i) to (iv) comment on how his / her findings make sense in real life situation.	The student **correctly but briefly explains** whether his or her results make sense in the context of the problem. The student **describes the importance** of his or her findings in connection to real life where appropriate. The student **attempts to justify** the degree of accuracy of his or her results where appropriate.
5–6	In question (d), the student is able to recognize and explain the applications seen in the vector operations. In question (e), the student has explained how this answer makes sense in the context of this problem.	The student **critically explains** whether his or her results make sense in the context of the problem. The student provides a **detailed explanation** of the importance of his or her findings in connection to real life. The student **justifies the degree of accuracy** of his or her results where appropriate. The student suggests improvements to his or her method where appropriate.

Student's self-evaluation
(0-6)
Teacher's Final Grade
(0-6)

A Special Matrix

Part 1

This section is assessed against criterion B only
You should spend 45-50 minutes on this section, and you are advised to make full use of your GDC.

Consider the following matrices:

$L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $M = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

(a) Find the matrix L^2

$$L^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 4+1 & 2+2 \\ 2+2 & 1+4 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} //$$

(b) Find the matrix M^2

$$M^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 9+4 & 6+6 \\ 6+6 & 4+9 \end{pmatrix} = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix} //$$

(c) Find the matrix N^2

$$N^2 = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 16+9 & 12+12 \\ 12+12 & 9+16 \end{pmatrix} = \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix} //$$

A student, Xavi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ then A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(d) Find A^2

You can't assume $A^2 = \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

$A^2 = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$= \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ //

Let a be 2,

$A = \begin{pmatrix} 2+1 & 2 \\ 2 & 2+1 \end{pmatrix}$

$= \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ // $= \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$A^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9+4 & 6+6 \\ 6+6 & 4+9 \end{pmatrix}$

$= \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$ // $= \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ //

$\because b = 12$

$\therefore A^2 = \begin{pmatrix} 12+1 & 12 \\ 12 & 12+1 \end{pmatrix} = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$ //

(e) Is Xavi right?

According to the calculations in part d, it is proven that Xavi is right.

$\because$ when $a = 2$, $A = \begin{pmatrix} 2+1 & 2 \\ 2 & 2+1 \end{pmatrix}$

$\therefore$ when $A^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$

$\because b = 12$

$\therefore A^2 = \begin{pmatrix} 12+1 & 12 \\ 12 & 12+1 \end{pmatrix}$ equals to the form by Xavi $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ //.

He is right because when he is got the form of $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$, when he put A to the power of 2 with the example above, it shows that A^2 realy equals to $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ //.

f-t.

(f) If your answer to **(e)** is "yes", is Xavi ***always*** right? Explain.
If your answer to **(e)** is "no", why is he wrong? Explain.

My answer to part is yes and Xavi will always be right because what he has created is the general form for A^2. The formula that Xavi generalized / guessed is the general formula for A and A^2, so no matter what numbers you put into A, the answer will still be $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ and $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ respectively.

f-t.

(g) Find the matrix LM

$$LM = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 6+2 & 4+3 \\ 3+4 & 2+6 \end{pmatrix}$$
$$= \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}_{//}$$

(h) Find the matrix MN

$$MN = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 12+6 & 9+8 \\ 8+9 & 6+12 \end{pmatrix}$$
$$= \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}_{//}$$

(i) Find the matrix LN

$$LN = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 8+3 & 6+4 \\ 4+6 & 3+8 \end{pmatrix}$$
$$= \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}_{//}$$

Another student, Messi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ and B is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

then the matrix AB is always of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(j) Prove (or disprove) Messi's hypothesis

$$AB = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix} = \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$$

~~Let, 1 = d~~

$$A = \begin{pmatrix} a+d & a \\ a & a+d \end{pmatrix}$$

$$B = \begin{pmatrix} b+d & b \\ b & b+d \end{pmatrix}$$

$$AB = \begin{pmatrix} a+d & a \\ a & a+d \end{pmatrix} \begin{pmatrix} b+d & b \\ b & b+d \end{pmatrix}$$

$$= \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$$

$\therefore$ AB is always in the form of $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

you can't assume this !

Now go back to the original matrices.

(k) Find L^3

$$L^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4+1 & 2+2 \\ 2+2 & 1+4 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 10+4 & 5+8 \\ 8+5 & 4+10 \end{pmatrix} = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix} //$$

(l) Find L^4

$$L^4 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\because L^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

$$\therefore L^4 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 25+16 & 20+20 \\ 20+20 & 16+25 \end{pmatrix}$$

$$= \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix} //$$

(m) Find L^5

$$L^5 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\because L^4 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$$

$$\therefore L^5 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 82+40 & 41+80 \\ 80+41 & 40+82 \end{pmatrix}$$

$$= \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix} //$$

(n) What do you notice about the form of the answers to (k), (l) and (m)? Try to use the various answers you have in order to generalize.

I have noticed that when the power of L increases, it is just multiplying another L to the answer from the previous power of L if the power of only increases by 1.

$\boxed{☆ L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}}$

When $L^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, it is actually $L^2 \times L$, and the answer will be the same as multiplying $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 3 times, and the answer is $\begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$.

In part (l), when L is to the power of 4, it only means $L^3 \times L$, which gives the answer $\begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$.

In part (m), when L is to the power of 5, it means $L^4 \times L$, which is $\begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and it will gives us the same answer as to multiply $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 5 times. The answer is $\begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$.

By looking at the previous 3 examples, we can generalize a formula which is $L^n = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^n$ and when the power is increased by 1, the form will be $L^{n+1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{n+1}$ ".

If we want find $L6$, it means $L^{5+1} = L^5 \times L = L^5 \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$$\begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 365 & 364 \\ 364 & 365 \end{pmatrix}.$$

This proofs that ~~the~~ rul $L^n = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^n$ and $L^{n+1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{n+1}$ is correct.

Bad guy runs and cop chases... "*Catch me if you can!*"

Part 2

This section is assessed against criterion D only. You should spend 35-40 minutes on this section.

One day, a thief (T) stole a handbag from a lady (L) , the lady shouted for help, a cop (C) was nearby, he then ran after the thief.....

It is given that the position of T, C and L are (3, 14), (1, 8) and (k, -2k) respectively, where k is a constant.

(a) (i) Express $\vec{CT}$ in terms of **i** and **j**.

$C = (1, 8) \quad T = (3, 14)$

$\vec{CT} = (3-1)\vec{i} + (14-8)\vec{j}$

$= 2\vec{i} + 6\vec{j}$ //.

(ii) Express $\vec{TL}$ in terms of **i**, **j** and **k**.

$C = (1, 8) \quad L = (k, -2k)$

$\vec{TL} = \vec{TC} + \vec{CL}$

$= 2\vec{i} + 6\vec{j} + \vec{CL}$

$= 2\vec{i} + 6\vec{j} + (k-1)\vec{j} + (-2k-8)\vec{k}$

$= 2\vec{i} + (6+k-1)\vec{j} + (2k-8)\vec{k}$ //.

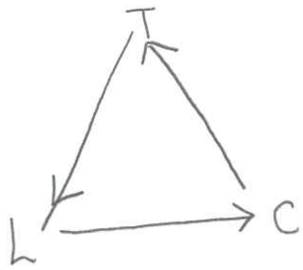
(b) If the value of k is -1, describe the meaning of this situation in real life. Show your work briefly.

If $k = -1$,

then $\vec{TL} = 2\vec{i} + (6-1-1)\vec{j} + ((-2)(-1)-8)\vec{k}$

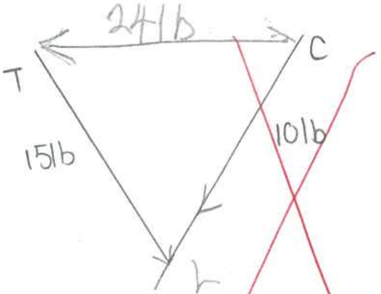
$= 2\vec{i} + 4\vec{j} - 6\vec{k}$.

When $k = -1$, this situation in real life means that you are not moving forward but actually being pushed backwards.



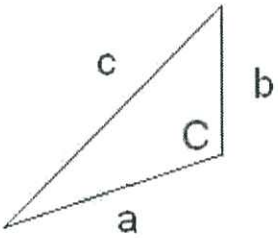
A few minutes later, T and C met… they started fighting for the handbag. Two forces with magnitudes of 15 lb and 10 lb were applied to the bag. The magnitude of the resultant was 24 lb.

(c) (i) Sketch a diagram to show each force in component form and the resultant force.



(ii) Find the measurement of the angle between the resultant vector and the vector of the 10 lb force to the nearest degree.

Hint: use cosine rule $\boxed{c^2=a^2+b^2-2(a)(b)(\cos C)}$



$c^2 = a^2 + b^2 - 2(a)(b)(\cos C)$

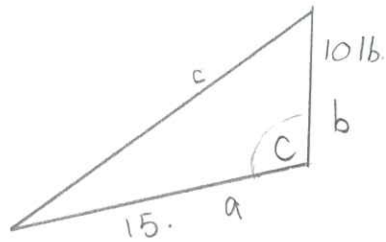
$c^2 = 15^2 + 10^2 - 2(15)(10)(\cos C)$

$c^2 = 325 - 300(\cos C)$

$\cos C = \sqrt{325 - 300}$

$\cos C = 5$

$C = 0.996°...$



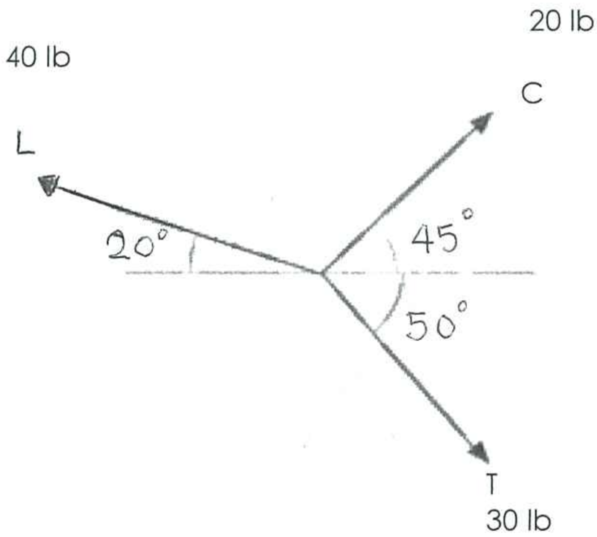
(iii) According to the above situation, the two forces from different directions are applied to the handbag, can the magnitude of the resultant be larger than the sum of the magnitudes of the forces? Justify your answer.

The magnitude of the resultant cannot be larger than the sum of the magnitudes of the forces because magnitude of the resultant is not the velocity of the resultant.

(iv) What if the forces were from the same direction? How would it affect the magnitude of the resultant force? Does it make sense in real life? Briefly justify your answer.

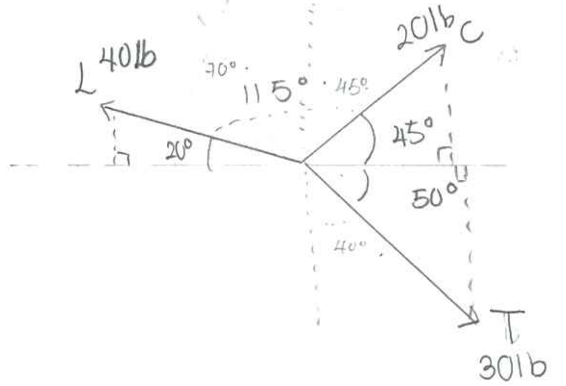
If the forces were from the same direction, the magnitude of the resultant will be affected because it affects the speed of the force.

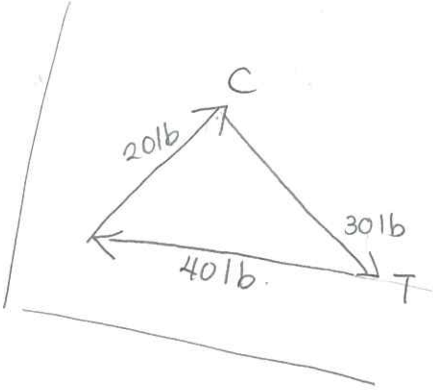
Finally, L caught up with T and C, she joined the fight as well. The diagram shows the 3 forces.



(d) The three forces shown in the diagram act at a point. Find the magnitude of their resultant and draw a diagram to show its directions. Who would have greater chance to get the handbag?

S A
T C



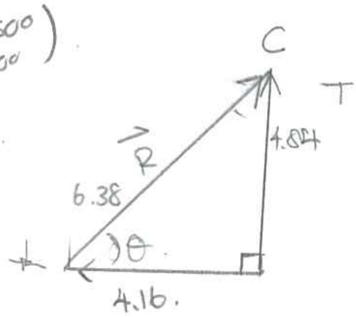


$C = \begin{pmatrix} 20 \cos 45° \\ 20 \sin 45° \end{pmatrix}$ $L = \begin{pmatrix} -40 \cos 20° \\ 40 \sin 20° \end{pmatrix}$ $T = \begin{pmatrix} 30 \cos 50° \\ -30 \sin 50° \end{pmatrix}$

$\begin{pmatrix} 20 \cos 45° \\ 20 \sin 45° \end{pmatrix} + \begin{pmatrix} -40 \cos 20° \\ 40 \sin 20° \end{pmatrix} + \begin{pmatrix} 30 \cos 50° \\ -30 \sin 50° \end{pmatrix}$

$= \begin{pmatrix} -4.16 \\ 4.84 \end{pmatrix}$ //

$\theta = 49.3.$



$\vec{R} = \sqrt{(4.84)^2 + (4.16)^2}$

$= 6.38$ ///

The direction is N 40.7° E.

The one who would have the greater chance to get the handbag would be the cop.

(e) Explain how this answer makes sense in the context of this problem.

The answer make sense in the context of this problem since the cop applied the most force when chasing the hand bag of the lady, so then he would have the greater chance in getting the handbag from the thief.

End of Assessment